Boolean rules

A+1=1	**A.0=0**
A+A=A	**A.A=A**
A+A′=1	**A.A′=0**
A+0=A	**A.1=A**
A+AB=A	**A.(A+B)=A**
A+A′B=A+B	**A.(A′+B)=AB**

Prove the identity **AB+A′C+BC=AB+A′C**

Proof: AB+A′C+BC = AB+A′C+BC.1
= AB+A′C+BC(A+A′) (A+A′=1)
= AB+A′C+BCA+BCA′
=AB(1+C) + A′C(1+B)
=AB+A′C (1+C=1, 1+B=1)

Prove the identity **(A+B)(A′+C)(B+C)=(A+B)(A′+C)**

Proof: (A+B)(A′+C)(B+C) = (A+B)(A′+C)(B+C+AA′) (AA′=0)
= (A+B)(A′+C)(B+C+A)(B+C+A′)
{Here, we have used the distributive property, A+BC=(A+B)(A+C)}
= (A+B)(A+B+C)(A′+C)(A′+C=B)
= (A+B)(A′+C)
{Here, we have used the absorptive law, A.(A+B)=A

Prove the identity, **(A+B)(A+B′)(A′+C)=AC**
Proof: (A+B)(A+B′)(A′+C) = (AA+AB′+AB+BB′)(A′+C)
= (A+AB+AB′)(A′+C) (BB′=0)
= {A(1+B)+AB′}(A′+C)
= (A+AB′)(A′+C) (1+B′=1)
= A(1+B′)(A′+C)
= A(A′+C)= AA′+AC = AC (AA′=0)

Q =	**(A + B).(A + C)**	
	A.A + A.C + A.B + B.C	– Distributive law
	A + A.C + A.B + B.C	– Idempotent AND law (A.A = A)
	A(1 + C) + A.B + B.C	– Distributive law
	A.1 + A.B + B.C	– Identity OR law (1 + C = 1)
	A(1 + B) + B.C	– Distributive law
	A.1 + B.C	– Identity OR law (1 + B = 1)
Q =	**A + (B.C)**	– Identity AND law (A.1 = A)

Example *Simplify the Boolean function F = AB + (AC)′ + AB′C(AB + C).*
= AB + A′ + C′+ AB′C.AB + AB′C.C
= AB + A′ + C′ + 0 + AB′C (B.B′ = 0 and C.C = C)
= ABC + ABC′ + A′ + C′ + AB′C (AB = AB(C + C′) = ABC + ABC′)
= AC(B + B′) + C′(AB + 1) + A′
= AC + C′+A′ (B + B′ = 1 and AB + 1 = 1)
= AC + (AC)′
= 1

Exercises:

Simplify the expression: **Y={(AB'+ABC)'+A(B+AB')}'**

Simply the expression and draw its logic circuit **Y=AB'+(A'+B)C**

Simplify the following logic expression **Y=(A'+B)(A+B)**